NUMERICAL SIMULATION OF STEEL CATENARY RISER

Marcus V. S. Casagrande
marcuscasagrande@coc.ufrj.br

Carlos E. Silva
José L. D. Alves
LAMCE/COPPE/UFRJ - BRAZIL
Av. Athos da Silveira Ramos, 149, Centro de Tecnologia - Bloco I - Sala 214, Cidade Universitária, Rio de Janeiro, 21941-909, RJ, Brasil

Abstract. The design of a steel catenary riser (SCR) is a challenging problem. The current design practice finds to overestimate forces on the riser due to limitations inherent to the simple soil usually employed. The principal objective of this investigation is to develop a model to simulate the behavior of a SCR riser. A 2D nonlinear beam element was implemented to represent the SCR riser.

Keywords: Steel catenary riser, Geometric nonlinear, Bidimensional beam
1 INTRODUCTION

The use of steel catenary risers (SCRs) has increased significantly in recent years with oil and gas productions moving to deep and ultra-deep waters. In such large depths, catenary risers are an important solution due to their lower installation cost. Usually, the motion of the floating vessel imposes an excitation at the top of the SCR, that is one of the main causes of the general dynamic response of the riser. Therefore, the dynamic analysis of the riser is a key issue an offshore floating unit that requires a proper formulation of the theoretical model to correctly represent the response (Chatjigeorgiou, 2008).

One of the critical issues in the design of a SCR is to estimate its fatigue life near its TDZ that is constrained by seabed soil. Fatigue is a progressive and localized structural damage that occurs when an object is subjected to cyclic loadings. Fatigue in SCR results mainly from the variation of the bending moment along the riser due to its oscillatory motion. According to Bhat (2004), the most probable localization of fatigue damage are near upper end where the riser is attached to the floating structure and also the TDZ where it contacts the seabed.

The proposed methodology of this work is based on the finite elements method (FEM). A bidimensional beam element with large displacement is presented. The Newmark method is the numerical scheme for the integration in the time domain (O'Brien et al., 1989).

2 MATHEMATICAL FORMULATION

The 2D nonlinear dynamic problem of a beam is considered. Due to large displacements of the riser, a geometric nonlinear element is utilized.

2.1 The Newmark method

According to Hughes (2012), the most widely used family of direct methods for solving the equation of motion (Eqs. (1), (2) and (3)) is the Newmark family. It consists of a method of numerical integration in the time domain.

\[ M \ddot{u} + C \dot{u} + Ku = F \]  
\[ u(0) = u_0 \]  
\[ \dot{u}(0) = v_0 \]

where \( M \) is the mass matrix, \( C \) is the viscous damping matrix, \( K \) is the stiffness matrix, \( F \) is the vector of applied forces, \( u \) is the displacement vector, \( \dot{u} \) is the velocity vector and \( \ddot{u} \) is the acceleration vector. The matrices \( M, C \) and \( K \) are supposed to be symmetric; \( M \) is positive-definite, and \( C \) and \( K \) are positive semi-definite.

Using finite difference formulas to describe the evolution of the approximate solution, the equations of Newmark family are presented in Eqs. (4), (5) and (6).

\[ Ma_{t+\Delta t} + C v_{t+\Delta t} + Ku_{t+\Delta t} = F_{t+\Delta t} \]  
\[ u_0 \]  
\[ v_0 \]
\[ u_{t+\Delta t} = u_t + \Delta t \, v_t + \frac{\Delta t^2}{2} \left[ (1 - 2\beta) a_t + 2\beta a_{t+\Delta t} \right] \]  

(5)

\[ v_{t+\Delta t} = v_t + \Delta t \left[ (1 - \gamma) a_t + \gamma a_{t+\Delta t} \right] \]  

(6)

where \( u_t, v_t \) and \( a_t \) are the approximations of the displacement, velocity and acceleration vectors at time \( t \). The parameters \( \gamma \) and \( \beta \) determine the stability and accuracy characteristics of the algorithm.

Usually the Eqs. (5) and (6) are inserted in Eq. (4) to calculate the only unknown \( a_{t+\Delta t} \) and subsequently \( u_{t+\Delta t} \) and \( v_{t+\Delta t} \) are calculated. This implementation is also called the \( a \)-form.

However, non-linear problems are normally solved in \( u \)-form. Equations (5) and (6) can be rewritten isolating the terms relative to the velocity and acceleration, resulting in:

\[ v_{t+\Delta t} = \frac{\gamma}{\beta} \Delta t \left( u_{t+\Delta t} - u_t \right) - \left( \frac{\gamma}{\beta} - 1 \right) v_t - \left( \frac{\gamma}{2\beta} - 1 \right) \Delta t \, a_t \]  

(7)

\[ a_{t+\Delta t} = \frac{1}{\beta \Delta t^2} \left( u_{t+\Delta t} - u_t \right) - \frac{1}{\beta \Delta t} \left( v_t - \left( \frac{1}{2\beta} - 1 \right) a_t \right) \]  

(8)

Once the velocity and acceleration are written as a function of the displacement, the equation of motion can be solved for the displacement at time \( t+\Delta t \), and subsequently the velocity and acceleration.

The basic problem in a general nonlinear dynamic analysis is to find the solution of the equation of motion according to the applied loads:

\[ M a_{t+\Delta t} = R_{t+\Delta t} - F_{t+\Delta t} \]  

(9)

where \( R_{t+\Delta t} \) is the vector of externally applied nodal forces at time \( t \) and \( F_{t+\Delta t} \) is the vector of internal forces in the configuration of time \( t \). For an iterative solution (Bathe, 2006), Eq. (9) has the form:

\[ M a^{(k)}_{t+\Delta t} = R_{t+\Delta t} - F^{(k)}_{t+\Delta t} \]  

(10)

The superscript indicates that the equation is being evaluated at iteration \( k \). In this equation, the external force do not have the superscript since it do not depend on the iteration. For consistency, a null superscript indicates that the variable should be evaluated at the last time step, ie, at time \( t \).

The vector of internal forces is a nonlinear function of the displacement. To linearize this vector, it is used the last internal force evaluated, and the vector is linearized at that point.

\[ F^{(k)}_{t+\Delta t} = F^{(k-1)}_{t+\Delta t} + \Delta F^{(k)} \]  

(11)

\[ \Delta F^{(k)} = K^{(k-1)}_{t+\Delta t} \Delta u^{(k)} \]  

(12)

\[ K^{(k-1)}_{t+\Delta t} = \frac{\partial F^{(k-1)}_{t+\Delta t}}{\partial u^{(k-1)}_{t+\Delta t}} \]  

(13)

where \( K^{(k-1)}_{t+\Delta t} \) is the tangent stiffness matrix which corresponds to the derivative of the internal force \( F^{(k-1)}_{t+\Delta t} \) with respect to the nodal point displacement \( u^{(k-1)}_{t+\Delta t} \) at the last available result. Then, Eq. (10) can be rewritten as:
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\[ M \ddot{u}^{(k)} + C \dot{u}^{(k)} + K u^{(k)} = F^{(k)} \]

(14)

Including a viscous force and rearranging the terms, Eq. (14) turns in:

\[ M \ddot{u}^{(k)} + C \dot{u}^{(k)} + K^{(k-1)} \Delta u^{(k)} = \dot{F}^{(k-1)} - R^{(k-1)} \Delta t \]

(15)

Using the Newton-Raphson method to find the solution of Eq. (15), the displacement at the current iteration \( k \), \( u^{(k)}_{t+\Delta t} \), is decomposed into the displacement at the previous iteration \( k-1 \), \( u^{(k-1)}_{t+\Delta t} \), and a variation of the displacement, \( \Delta u^{(k)} \):

\[ u^{(k)}_{t+\Delta t} = u^{(k-1)}_{t+\Delta t} + \Delta u^{(k)} \]

(16)

Implementing the Newton-Raphson method in the Newmark family equations, the following equations are obtained:

\[ v^{(k)}_{t+\Delta t} = \frac{r}{\beta \Delta t} \left( u^{(k-1)}_{t+\Delta t} + \Delta u^{(k)} - u_t \right) - \left( \frac{r}{\beta} - 1 \right) \dot{v}_t - \left( \frac{1}{2\beta^2} - 1 \right) \Delta t a_t \]

(17)

\[ a^{(k)}_{t+\Delta t} = \frac{1}{\beta \Delta t^2} \left( u^{(k-1)}_{t+\Delta t} + \Delta u^{(k)} - u_t \right) - \frac{1}{\beta \Delta t} \dot{v}_t - \left( \frac{1}{2\beta} - 1 \right) a_t \]

(18)

Substituting Eqs. (17) and (18) in Eq. (15), and selecting the variation of the displacement as the main unknown, the final equation can be written in a reduced form as:

\[ \hat{R}^{(k-1)} \Delta u^{(k)} = R^{(k-1)} - \hat{F}^{(k-1)}_{t+\Delta t} \]

(19)

where \( \hat{R}^{(k-1)}_{t+\Delta t} \) is the effective tangent stiffness matrix and \( \hat{F}^{(k-1)}_{t+\Delta t} \) is the effective internal force vector.

\[ \hat{R}^{(k-1)}_{t+\Delta t} = \frac{1}{\beta \Delta t^2} M + \frac{r}{\beta \Delta t} C + K^{(k-1)}_{t+\Delta t} \]

(20)

\[ \hat{F}^{(k-1)}_{t+\Delta t} = F^{(k-1)}_{t+\Delta t} + M \ddot{u}^{(k-1)}_{t+\Delta t} + C \dot{v}^{(k-1)}_{t+\Delta t} \]

(21)

The terms \( \hat{u}^{(k-1)}_{t+\Delta t} \) and \( \hat{a}^{(k-1)}_{t+\Delta t} \) are also called the predicted velocity and acceleration.

\[ \hat{v}^{(k-1)}_{t+\Delta t} = \frac{r}{\beta \Delta t} \left( u^{(k-1)}_{t+\Delta t} - u_t \right) - \left( \frac{r}{\beta} - 1 \right) \dot{v}_t - \left( \frac{1}{2\beta^2} - 1 \right) \Delta t a_t \]

(22)

\[ \hat{a}^{(k-1)}_{t+\Delta t} = \frac{1}{\beta \Delta t^2} \left( u^{(k-1)}_{t+\Delta t} - u_t \right) - \frac{1}{\beta \Delta t} \dot{v}_t - \left( \frac{1}{2\beta} - 1 \right) a_t \]

(23)

The solution of the system can be found inverting the effective tangent stiffness matrix and solving Eq. (19) for the variation of the displacement. To complete the time cycle the displacement, velocity and acceleration are updated via Eqs. (9), (24) and (25).

\[ v^{(k)}_{t+\Delta t} = \hat{v}^{(k-1)}_{t+\Delta t} + \frac{r}{\beta \Delta t} \Delta u^{(k)} \]

(24)

\[ a^{(k)}_{t+\Delta t} = \hat{a}^{(k-1)}_{t+\Delta t} + \frac{1}{\beta \Delta t^2} \Delta u^{(k)} \]

(25)

The mainstream of the code is:

I) Apply initial conditions \( (u_0; v_0; a_0) \)

II) Initialize auxiliary vectors \( (u0 = v0 = a0 = vpred = apred = 0) \)
III) $t=0$

IV) TIME LOOP (N=1..NSTEPS)

i) $t = t + \Delta t$

ii) Update external force ($F$)

iii) $u_0 = u_n$

iv) Compute $v_{pred}$ and $a_{pred}$

   - $v_{pred} = c_1*(u_n-u_0) - c_2*v_0 - c_3*a_0$
   - $a_{pred} = c_4*(u_n-u_0) - c_5*v_0 - c_6*a_0$

v) Compute internal force $F_{int} = f(u_n)$

vi) Compute the equivalent internal force $F_{eq}$

   - $F_{eq} = F_{int} + M*a_{pred} + C*v_{pred}$

vii) Iteration loop

   - Compute $K = f(u_n)$
   - Solve the system $du = K^{-1}[F_{eq}]$
   - Update displacement: $u_n = u_n + du$
   - Update $v_{pred}$ and $a_{pred}$
     - $v_{pred} = v_{pred} + c_7*du$
     - $a_{pred} = a_{pred} + c_8*du$
   - Update $F_{eq}$
   - Update $F$

   - Compute de residue’s norm: $res = || F_{eq} ||$
   - Iteration stop condition
   - Restart iteration loop

viii) Compute $v_n$ and $a_n$

   - $v_n = v_{pred} + c_7*du$
   - $a_n = a_{pred} + c_8*du$

V) END TIME LOOP

where $u_n$, $v_n$, $a_n$ are the solution for displacement, velocity and acceleration at current time ($t+\Delta t$) ($u_n$, $v_n$, $a_n$); $u_0$, $v_0$, $a_0$ are the displacement, velocity and acceleration of the last time step ($t$) ($u_t$, $v_t$, $a_t$); $v_{pred}$ and $a_{pred}$ are the predicted velocity and acceleration ($v_{pred}^{(k-1)}$, $a_{pred}^{(k-1)}$); $F$ is the external force vector ($R_{t+\Delta t}$), $K$ is the effective tangent stiffness
matrix \( \mathbf{R}^{(k-1)}_{t+t\Delta t} \), \( \mathbf{F}_{t}^{(k-1)} \) is the internal force vector, \( \mathbf{F}_{t+t\Delta t}^{(k-1)} \) is the effective internal force vector, and \( \mathbf{F}_{t+t\Delta t}^{\text{inteq}} \) is the internal force vector. The constants \( c_1, c_2, c_3, c_4, c_5, c_6, c_7 \) and \( c_8 \) are in accordance with the equations presented in this section.

### 2.2 Total Lagrangian method

The total Lagrangian method is appropriate for large rotations and small strains, which is the case of the risers. When large deflections are present, the equations of force equilibrium must be formulated for the deformed configuration of the element. To account for the effects of changes in the geometry as the external force is applied, the solution of the nonlinear problem in a sequence of linear steps, each step representing a load step or a time step. However, because of the presence of large-deflections, strain-displacement equations contain nonlinear terms, which must be included in calculating the stiffness matrix (Przemieniecki, 1985).

The nonlinear terms in the strain-displacement equation modify the element stiffness matrix \( \mathbf{K}^e \) so that

\[
\mathbf{K}^e = \mathbf{K}^e_E + \mathbf{K}^e_G
\]

where \( \mathbf{K}^e_E \) is the linear elastic stiffness matrix calculated for the element at the start of each iteration, and \( \mathbf{K}^e_G \) is the geometrical stiffness matrix.

\[
\mathbf{K}^e_E = \frac{E I}{l^3} \begin{bmatrix}
\frac{AL^2}{l} & 0 & 12 & 4L^2 \\
0 & 6L & 4L^2 & 0 \\
-\frac{AL^2}{l} & 0 & 0 & \frac{AL^2}{l} \\
0 & -12L & -6L & 0 & 12 \\
0 & 6L & 2L^2 & 0 & -6L & 4L^2
\end{bmatrix}
\]

\[
\mathbf{K}^e_G = \frac{N}{L} \begin{bmatrix}
0 & 0 & \frac{6}{5} & \frac{2}{15}L^2 \\
0 & \frac{L}{10} & \frac{2}{15}L^2 & 0 \\
0 & -\frac{6}{5} & -\frac{L}{10} & 0 & \frac{6}{5} \\
0 & \frac{L}{10} & -\frac{L^2}{30} & 0 & -\frac{L}{10} & \frac{2}{15}L^2
\end{bmatrix}
\]

where \( L \) is the element length, \( I \) the moment of inertia, \( E \) the modulus of elasticity, and \( N \) the element axial internal force.

The rigid body motion in finite elements can create residual stress due to the large-displacements of the element. To outline this issue, it is defined a natural displacement vector \( \mathbf{u}^e_G \). This natural displacement in the total lagrangian method represents the stress-related
displacements in a local referential. For each element, the global displacement vector is defined as:

\[
\mathbf{u}^e = \begin{bmatrix} u_1 \\
v_1 \\
\theta_1 \\
u_2 \\
v_2 \\
\theta_2 \end{bmatrix}
\]  \hspace{1cm} (29)

where \( u_i \) is the displacement in horizontal direction of node \( i \), \( v_i \) is the displacement in vertical direction of node \( i \), and \( \theta_i \) is the angular displacement of node \( i \). The local referential is defined with the origin at node 1, so there is no horizontal or vertical displacement at this node, and the first principal direction passes through the node 2, that causes no vertical displacement at node 2. For each element, the natural displacement vector is defined as:

\[
\mathbf{u}^e_n = \begin{bmatrix} 0 \\
0 \\
\varphi_1 \\
\delta \\
0 \\
\varphi_2 \end{bmatrix}
\]  \hspace{1cm} (30)

where,

\[
\delta = L_{t+\Delta t} - L_{\text{initial}} \hspace{1cm} (31)
\]

\[
\varphi_1 = \theta_1 - \Psi \hspace{1cm} (32)
\]

\[
\varphi_2 = \theta_2 - \Psi \hspace{1cm} (33)
\]

\[
v = v_2 - v_1 \hspace{1cm} (34)
\]

\[
u = u_2 - u_1 \hspace{1cm} (35)
\]

\[
\Psi = \tan^{-1}\left(\frac{v}{t+\Delta t + u}\right) \hspace{1cm} (36)
\]

However, the stiffness matrix of the element is defined in the local referential. To transform the local to global referential, it is defined the rotation matrix:

\[
\mathbf{R}^e = \begin{bmatrix}
\cos(\Psi) & \sin(\Psi) & 0 & 0 & 0 & 0 \\
-\sin(\Psi) & \cos(\Psi) & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \cos(\Psi) & \sin(\Psi) & 0 \\
0 & 0 & 0 & -\sin(\Psi) & \cos(\Psi) & 0 \\
0 & 0 & 0 & 0 & 0 & 1 
\end{bmatrix}
\]  \hspace{1cm} (37)

The element internal force vector \( \mathbf{F}^e \) can be evaluated through the natural displacement vector \( \mathbf{u}^e_n \) and the linear stiffness matrix \( \mathbf{K}^e \).

\[
\mathbf{F}^e = [\mathbf{R}^e^T \mathbf{K}^e \mathbf{R}^e] \mathbf{u}^e_n
\]  \hspace{1cm} (38)
The internal force vector $F^{(k-1)}_{t+\Delta t}$ is the assembly of all element internal force vectors. Analogously, the tangent stiffness matrix $K^{(k-1)}_{t+\Delta t}$ is the assembly of all element stiffness matrix rotated to the global coordinate of system $[R^e]^T K^e R^e]$.

Finally, the element mass matrix $M^e$ is defined as:

$$
M^e = \frac{\rho A L}{420} \begin{bmatrix}
140 & 156 & \text{Sym.} \\
0 & 22L & 4L^2 \\
70 & 0 & 0 & 140 \\
0 & 54 & 13L & 0 & 156 \\
0 & -13L & -3L^2 & 0 & -22L & 4L^2
\end{bmatrix}
$$

(39)

3 STUDIED CASE

Riser structures can usually be modelled as cables, which have catenary shapes as equilibrium solutions. However the bending stiffness starts to play an important role when small curvatures are present.

In the addressed case, the riser is modelled with the bidimensional nonlinear beam element described in this work. The purpose of this example is an initial validation of the code. In this analysis, the inertial effects are neglected. However, the quasistatic analysis uses the same incremental and interactive solution described previously.

The considered structure have simply supported boundary conditions at both ends. The riser is divided into 100 elements and has an initial length of $L = 142.5m$. The main properties considered are the cross sectional area $A = 8 \times 10^{-3} m^2$, moment of inertia $J = 5.1 \times 10^{-6} m^4$ and modulus of elasticity $E = 100.0 GPa$ (Hosseini Kordkheili, 2011).

Initially, it is calculated the equilibrium position of the riser with both ends simply supported and the gravity acting on it. After the initial configuration is achieved, a quasistatic analysis is conducted to move the right end of the riser horizontally to near the left end by 112.5m.

This procedure was realized utilizing the implemented element, and utilizing Ansys software with a similar nonlinear element to compare the results. The results are presented in Figure 1 through five different configurations: the initial equilibrium conditions and the right boundary movement of 12.5m, 42.5m, 82.5m and 112.5m. It can be seeing an excellent agreement between both nonlinear finite element solutions.
Figure 1: Comparison of several configurations of the riser with the right end moving to the left between the implemented element (solid line) and Ansys software (symbols).

From the last riser configuration exhibited at Figure 1, it is noticed an expressive difference between the catenary solution and the beam solution, since there is a negative horizontal position of the riser. In the catenary solution this behavior is not present, and it is due to the flexural stiffness of the beam element.

FINAL REMARKS

This work presented the main aspects of an implementation of a bidimensional beam element with geometric nonlinearity.

The studied case showed excellent agreement between a commercial software and the developed code as an initial verification of the algorithm. For future work we intend to extend the verification of the implementation to dynamic problems and latter a tridimensional formulation.

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