ABSTRACT: This paper deals with a numerical-computational model that simulates the dynamic behaviour of a train-railway system. This model considers the sleeper-ballast group as elastic supports and the loading from the train is evaluated along the railway. Aiming the evaluation of the proposed model, experimental tests on the MRS railway near the city of Barra do Piraí – RJ, Brazil, were performed by using a VHS camera. By means of image processing, camera images were used in order to evaluate vertical displacements of the railway, during a train traffic. The numerical-experimental comparisons showed that the obtained numerical results, present good qualitative agreement with the experimental data.

1. INTRODUCTION

The growing demands for transportation of grains, minerals, passengers and others reaffirm the importance of railways around the world as indicate the increasing investments in countries like France (Anonymous, 2009), USA (Barrow, 2008), China (Anonymous, 2009) and Scotland (Briginshaw, 2009). According to Rodrigues (2001), it is estimated to be applied in Brazil around U$150,000,000.00 per year in railway maintenance resulting in a cost of U$ 10,000.00 per kilometre per year in railroads maintenance. It represents around 15% to 30% of the total railway operation cost.

Besides economical importance involved in railroad maintenance, another motivation for researches in this area is related to user’s safety. There are reports of accidents due to rail fracture problems and also to low rigidity of rail-sleeper-ballast group. These kind of accidents could be avoided with the use of health monitoring techniques (da Silva, 2002). The following papers present some techniques used to prevent railway accidents and to reduce maintenance costs: Jaiswal (2005), Roden (2005) and Knutton (2004).

Computational models show that, in certain cases, it is possible to satisfactory simulate structural railroad behaviour with relatively simple considerations. Some models, as the one used in the present work, consider the loading from the train as a set of concentrated loads moving along a continuous beam on elastic supports. In this case, the beam represents the rail and the supports represent the sleeper-ballast group. In other models, the train may be considered as a set of supported mass on springs and a damper system simulate vehicle suspensions. More sophisticated models are presented by Kaewunruen & Remennikov (2007); Knothe et. al. (1994); Kumaran et. al. (2002) and Nielsen & Igeland (1995).These sophisticated models have received considerable attention in the development of high-speed trains (Lombaert & Degrand (2007)).

Usually, model validations are carried out through comparisons with in locu experimental measurements obtained by devices as extensometers and accelerometers or, alternatively, by techniques of image processing (Bownness et. al. (2007)) as will be shown in this work.

Numerical and experimental analysis of the train-railway interaction

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2. DESCRIPTION OF THE EMPLOYED COMPUTATIONAL MODEL

The model that was used in this work is based on the Finite Elements Method (FEM) and it allows the evaluation of railway vertical displacements due to train traffic. It considers the loading from the train as a set of concentrated loads moving along a continuous beam on elastic supports. Figure 1 shows the beam model which represents the rail supported by elastic springs. In this figure, $K$ represents the equivalent stiffness of the sleeper-ballast group. This model adopts a lumped mass discretization, being $m_i (i=1...50)$ the total mass of the $i$th element.

In the present work, the modelled rail was a TR68 (M.I.M.F, 2009) which presents: $E = 2.11 \times 10^{11}$ kN/m$^2$; $A = 3.15 \times 10^{-3}$ m$^2$; $I = 3.949 \times 10^{-5}$ m$^4$ and density $\gamma = 7850$ kg/m$^3$. The discretization of the rail was made with all elements having length equal to $L = 0.5$m, defined as the distance between two consecutive sleepers.

The railroad was modelled as a continuous beam using plane frame finite elements. These elements have the well known stiffness matrices $K_{el}$ as shown in equation (1), where $E$, $I$, $A$ and $L$ are respectively, the rail material Young’s modulus, the inertia momentum and the area of the rail cross section, and the element length.

$$K_e = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{-12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{-6EI}{L^2} & \frac{2EI}{L} \\ \frac{-EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & \frac{-12EI}{L^3} & \frac{-6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{-6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$  

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The stiffness matrix $K$ of the system is obtained by adding elementary contributions of $K_{el}$, as well as the rigidity contributions $K$ from the elastic supports (see Figure 1).

Lumped mass was used in the adopted model, resulting in global matrix ($M$) as shown in equation (2).

$$M = \text{diag} \left[ \frac{(m_1)}{2}, \frac{(m_1 + m_2)}{2}, ..., \frac{(m_{49} + m_{50})}{2}, \frac{m_{51}}{2} \right]$$

Figure 2(a) and Figure 2(b) present, respectively, a schematic drawing of two cars and the used model for the train. It is observed that the vehicle model considers each car axis as a concentrated load on the rail. In that way, there is no consideration of inertial forces from the train and the car damping system is not taken into account either.
The adopted model for the train allows the introduction of the following characteristics: amount of cars \( n = 30 \); velocity of the train \( V \); distance between trucks \( d_2 \); distance between cars \( d_3 \); load of front axis \( P_1 \); load of back axis \( P_2 \). Values of \( d_1 \), \( d_2 \) and \( d_3 \) were set as 1.778m, 6.20m and 4.20m, respectively, and loads \( P_1 \) and \( P_2 \) were both set as 116.49kN (typical values for the used train).

Using geometrical features of the cars and train speed, it is possible to calculate positions of all train axis and, consequently, equivalent nodal force vector \( \mathbf{F}_j \) for each axis \( j \), by regarding Figure 3 and using equation (3). The load vector \( \mathbf{F} \) of the system is evaluated for each analysis time step by the adequate summation of \( \mathbf{F}_j \).

![Figure 3. Frame Finite Element with a vertical force](image)

\[
\mathbf{F}_j = \begin{bmatrix} H_1 \\ V_1 \\ M_1 \\ H_2 \\ V_2 \\ M_2 \end{bmatrix} = \begin{bmatrix} 0 \\ L' - 3a^2L + 2a^3 \\ \left(L' - 2aL + a^2\right)\mu L \\ 0 \\ (3L - 2a)\alpha^2 \\ (a - L)a^2L \end{bmatrix}
\]

where: \( H_i, V_i, M_i \) \((i=1...2)\) represent, respectively, the \( i^{th} \) nodal horizontal and vertical forces and applied momentum; \( L \) is the length of the element; \( a \) is the distance from the load application point and element left node; \( P \) is vertical load of the \( j^{th} \) axis.

The structural response in terms of displacements \( \mathbf{X} \), velocities \( \dot{\mathbf{X}} \) and accelerations \( \ddot{\mathbf{X}} \) may be obtained by integrating the dynamic equilibrium equations described in equation (4). The used
integration method was Newmark (Bathe, 1996). In equation (4), \( C \) is the damping matrix considered as proportional to the mass.

\[
M\ddot{x} + C\dot{x} + Kx = F(t)
\] (4)

3. EXPERIMENTAL MEASUREMENTS

Experimental measurements of deflections in rails is traditionally carried out by using the Benkelman beam, schematically shown in Figure 4. Created by A. C. Benkelman, this beam is basically composed by a fixed part and a mobile rod. The fixed part is supported by two adjustable feet, rested on the ground in positions not affected by the soil movement caused by the train. The mobile rod is coupled to the fixed part through an articulation, being one of its ends (end of proof) in contact with the rail and the other end in contact with an extensometer. Using geometrical compatibility equations, it is possible to evaluate the vertical movement of the tip, by using extensometer measurements.

Another way to measure deflections is through laser based equipments. This measurement systems allow the evaluation of rail vertical displacements in high frequencies. Figure 5(a) and Figure 5(b) present, respectively, experimental test photos with a laser system and results for measurements made by Cibermétrica, 2007.

![Figure 4. Schema of a Benkelman beam](image)

(a) system of laser measurements

(b) Laser sensor

(c) Response obtained with a laser system (extracted from Cybermétrica, 2007)

Figure 5. Vertical displacement evaluation using a laser based equipment.
In the present study, vertical deflection measures of a rail were obtained by means of image processing from an experimental test. The dynamic behaviour of the vehicle-sleeper-ballast group excited by a train having GDT cars moving on rails TR68 (M.I.M.F. 2009) was analysed regarding the railroad of MRS Logistics near the city of Barra do Piraí – RJ, Brazil, shown in Figure 6.

The test layout is shown in Figure 7, where one observes a camera with tripod placed around six meters far from railroad track and a target with square shapes. The adopted distance between the camera and the railway were sufficient to avoid traffic perturbation in measurements (Cibermétrica,2007).

A VHS grayscale video was recorded during the passage of train cars. The recorded movie has an acquisition rate of 30 frames per second, having an image sequence of the target central square, as shown in Figure 8. The used target has various squares in order to allow the evaluation of rotations, although it is not focused herein.
For each frame of the image sequence, an automatic processing of the target is executed. The first step is to perform image binarization by means of a threshold procedure as shown in Figure 9.

The threshold limit $T$ was manually adopted, after the observation of the first movie frame pixel histogram, as presented in Figure 10. This value was considered constant during all the image processing.

For the pixels of the image $(x, y)$ that exceed the value of $T$, it is set $b(x, y)=1$ (black pixels) otherwise $b(x, y)=0$ (white pixels). By applying this procedure for each frame of the movie one may generate a binary frame sequence and, using equation (7) and equation (8), it is possible to calculate the centroid of each target central square.
\[
\bar{x} = \frac{1}{N} \sum_{y_1}^{y_N} \sum_{x_1}^{x_N} b(x, y)x
\]
(7)
\[
\bar{y} = \frac{1}{N} \sum_{x_1}^{x_N} \sum_{y_1}^{y_N} b(x, y)y
\]
(8)

where \(x_N\) and \(y_N\) are, respectively, the number of pixels in horizontal and vertical directions inside the target central square region, and \(N = x_N \cdot y_N\).

Finally, using known actual target dimensions and their correspondent image size obtained from the video frames, it is possible to establish a relationship between the recorded target movements and actual rail vertical displacements.

4. CALIBRATION OF THE NUMERICAL MODEL

Vehicle speed was obtained by analysing recorded images as it could not be directly measured during the experimental test. To this end, target brightness was used since it was drastically affected by the train axis shadows during the car passages over the target. By observing the darkest frames in the movie, and car geometry (see Figure 2) axes positions, and consequently train speed, were estimated.

As the proposed model does not consider vehicle acceleration, an average speed of 3.6 m/s was calculated and adopted as a constant.

The sleeper-ballast stiffness constant \(K\) (see Figure 1) was set as \(K = 3.8 \times 10^7\) N/m. It was observed that rail vertical displacement amplitudes are inversely proportional to the stiffness constant \(K\) as it can be noticed in Figure 11.

![Figure 11. Influence of the stiffness constant K on the rail vertical displacement amplitudes](image)

5. RESULTS

In spite of the lack of comparisons between the obtained experimental results with another method of measurement, as the Benkelman beam, several comparisons using synthetic videos were carried out. This results were presented in Nogueira (2006).

Figure 12 presents numerical and experimental system responses. Despite the simplicity of the used model, the numerical responses are in relatively good agreement with their experimental counterparts.
Differences between numerical and experimental results are probably due to the following features that were not taken into account in the numerical model: variations in train speed during the experimental test; the neglecting of train inertial forces; the disregard of local effects in the train-railway interactions.

![Figure 12: The obtained numerical and experimental results.](image)

Obviously, differences between experimental and numerical results may be also attributed to errors inherent to any experimental measurement.

6. CONCLUSIONS

Numerical and experimental results obtained in this work presented good correlation. In that way, it is reasonable to affirm that the used computational model may be used to estimate dynamical behaviour of vehicle-railroad interaction in terms of vertical rail displacements.

The used measurement methodology lacks validation tests as the comparison of their results with consolidated techniques as Benkelman beams. Nevertheless, obtained results indicate that the proposed technique may be a viable alternative for dynamic measurement in railroads.

Finally, the presented numerical-experimental strategy seem to be applicable as a low cost tool in the dynamical analysis of vehicle-railroad interaction.

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