Evaluation of structural natural frequencies using image processing

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ABSTRACT: In Structural Engineering, the evaluation of natural frequencies is very important since design codes recommendations are often based on frequency limits. Traditionally, natural frequencies of structures are evaluated from analogical or digital conditioned signals obtained with displacement, strain or acceleration transducers, which present several configurations and functional designs. Alternatively, one can use image-processing techniques to acquire structural dynamic behaviour and consequently, to evaluate natural frequencies. By means of digital image, it is possible to make accurate displacement measuring in structures avoiding the use of electronic transducers and signal conditioner, which are, in general, considerably more expensive than a standard camera. This method consists on: firstly, the movement of a black circle target on a white background fixed on the analysed structure is recorded; secondly, the target’s centroid of each digital image is computed (with sub-pixel precision), resulting in a set of spatial coordinates in function of time; finally, by means of applying time-frequency transform methods, the structural natural frequencies are obtained. Numerical simulations of the presented method are analysed, as well as an instrumented laboratory beam in order to assess its efficiency.

1 INTRODUCTION

The utilization of digital images in order to analyse dynamical behaviour of structures is present in some works in the literature [1,2]. Specifically, in the structural dynamic domain, the authors analyse, structural damage [1] and, in the most cases, modal identification [2] as the present work.

This article presents a very simple method used to identify natural frequencies using image processing. This method consists on: firstly, the movement of a black circle target on a white background fixed on the analysed structure is recorded; secondly, the target’s centroid of each digital image is computed (with sub-pixel precision), resulting in a set of spatial coordinates in function of time; finally, by means of applying time-frequency transform methods, the structural natural frequencies are obtained.

2 THE PROPOSED APPROACH

The proposed methodology presented in this work consists on to record a fixed target placed on a structure under free vibration using a digital camera. The obtained video is transferred to a computer and the developed software is applied in order to automatically identify the central target coordinates for each video frame. This process, after simple algebraic operations, results in displacement time series that may be transformed into frequency domain using Discrete Fourier Transform. The natural vibration frequencies of the structure may be directly extracted from the frequency domain response.
2.1 Model and image processing

This paper analyses the dynamic behaviour of a cantilever beam through natural frequency evaluation using image sequences of a target placed on the structure. Figure 1 presents the beam model with a circular target and also shows the global and the image coordinate systems adopted in this paper.

![Figure 1 - Cantilever beam model, target and coordinate systems.](image)

The camera position was defined by its perspective centre coordinates \( P_c(X_{Pc}, Y_{Pc}, Z_{Pc}) \) using global system as referential, having orientation defined by rotation angles \( \kappa \) (Z axis), \( \varphi \) (Y axis), and \( \theta \) (X axis).

The target must allow a robust identification through automatic methods and it should present geometric characteristics compatible to the application. In this way, a black circle on a white background is used as target.

A simple thresholding is capable to transform a grayscale (8 bits) or true colour (24 bits) image into a binary (1 bit) image as presented in figure 2 [3]. In this case a grayscale image is transformed into a binary image. The thresholding defines the limit between the light and dark image pixel grayscale, transforming light ones into white pixels, and dark ones into black pixels.

Using a binary image, the pixels attached to the black colour (inside the circle) have the label \( b(x,y)=1 \), and the pixels attached to the white colour (outside the circle) have the label \( b(x,y)=0 \), as shown in figure 2.
The thresholding limit may be considered constant if the illumination conditions are stable during all operation.

After the target pixel identification, it is possible to determine the black circle image centroid \((\bar{x}, \bar{y})\), shown in figure 2, with sub-pixel precision using:

\[
\bar{x} = \frac{1}{N \times M} \sum_{x=1}^{N} \sum_{y=1}^{M} b(x, y) x
\]  

(1)

and:

\[
\bar{y} = \frac{1}{N \times M} \sum_{x=1}^{N} \sum_{y=1}^{M} b(x, y) y
\]  

(2)

where:

- \(N\) and \(M\) are the number of columns and rows of the image, respectively.

Normalizing summation of \(b(x,y)\) to the unit, expressions (1) e (2) may be faced as the first probability distribution moment of \(b(x,y)\).

2.2 Time history series and natural frequencies

By taking the coordinates \((\bar{x}, \bar{y})\) obtained for each frame, it is possible to generate time history series \(\bar{x}_t\) and \(\bar{y}_t\), where \(t\) represents the frame number. These time series allow structural natural frequency identification that, in this work, is obtained by converting time response to frequency domain using Discrete Fourier Transform.

2.3 Limitations of the proposed method

This methodology is quite simple and relatively not expensive but it may be not generally applied due to some limitations. A modal identification using the proposed method may present errors in some particular situations.

The first limitation is attached to the aliasing problem. The standard digital camera used in the analysis has frame rate equals to 30 frames per second, and this situation demands a dynamic
behaviour with maximal frequency component of 15 Hz (Nyquist frequency). This low frequency limit may be increased using a higher frame rate camera, which is obviously more expensive than the used camera.

Secondly, it must be observed that images are bi-dimensional (2D) projections of the three-dimensional (3D) space. In that case, the perspective effects of this 3D to 2D transformation affect the natural frequency identification when the camera sensor plan is not parallel to the analysed structure oscillation plan. If this parallelism does not occur, the magnitude of the identified natural frequencies using the proposed method are changed and ghost frequencies appear in the time series \( x_t \) and \( y_t \). Moreover, displacement components perpendicular to the camera sensor plan are not easily detected and may also cause ghost frequencies. The sensibility of the frequency identification due this limitation is analysed in experimental test.

Finally, illumination and reverberation problems may cause inaccurate results. This kind of problem is inherent to image processing methods.

3 TESTS AND RESULTS

In order to assess the proposed method, synthetic videos were used simulating test results, as well as a real video obtained from experimental test with of a cantilever beam.

3.1 Synthetic videos

The synthetic videos were generated using Pov-Ray v3.6 software. Videos having circular black target with white background; 30 frames per second; 10 seconds of time length; and frame size of 160 lines per 120 columns, were created and analysed using the proposed method. The target centroid displacement was modelled by equations:

\[
X_t = \sum_{i=1}^{3} A_i \cos(2\omega_i t)e^{-\xi \omega_i t}
\]

(3)

\[
Y_t = \sum_{i=1}^{3} B_i \sin(\omega_i t)e^{-\xi \omega_i t}
\]

(4)

where:
- \( X_t \) and \( Y_t \) are the 3D coordinates in global system.
- \( A_i \), \( B_i \) and \( C \) are the frequency amplitudes;
- \( \omega_i = 2\pi f_i \) are angular frequencies in X and Y directions, being \( f_i \) in Hertz.
- \( \xi \) is the damping ratio, considered constant for all frequencies in this model.

Expressions (3) and (4) approach, respectively, the horizontal and the vertical displacements of the free extremity of a cantilever beam. It is important to notice that the simulated beam has oscillation frequency in X direction two times bigger than Y direction frequency counterpart. Displacements in Z direction were ignored.

Three synthetic video tests were development in order to assess the proposed method. Test #1 simulates ideal conditions, where camera sensor plan is parallel to the analysed structure oscillation plan. In test #2, small rotations \( \theta \) and \( \phi \) are applied to the camera sensor plan. Test #3, significant rotations \( \theta \), \( \phi \) and \( \kappa \) are imposed to the camera sensor plan. Tables 1, 2 and 3 describe the synthetic test parameters.
Table 1– Parameters of Test #1

<table>
<thead>
<tr>
<th>X_{pc}</th>
<th>f_1</th>
<th>ξ</th>
<th>A</th>
<th>B</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0°</td>
<td>2</td>
<td>0.25</td>
<td>A_1</td>
<td>0.1</td>
</tr>
<tr>
<td>Y_{pc}</td>
<td>f_2</td>
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<td>0.13</td>
<td>A_2</td>
<td>0.1</td>
</tr>
<tr>
<td>Z_{cp}</td>
<td>f_3</td>
<td>7</td>
<td>0.14</td>
<td>A_3</td>
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Table 2– Parameters of Test #2

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<th>B</th>
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<tbody>
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<td>f_3</td>
<td>7</td>
<td>0.14</td>
<td>A_3</td>
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Table 3– Parameters of Test #3

<table>
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<tr>
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<th>ξ</th>
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<tr>
<td>Z_{cp}</td>
<td>f_3</td>
<td>7</td>
<td>0.14</td>
<td>A_3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Figure 3 presents the identified time histories in X and Y directions for test #1. Time history identifications for the other tests present similar results and, for this reason, they are not presented herein. The displacement unit shown in figure 3 is “number of pixels”.

Figure 4 shows the frequency response for all synthetic video tests. For test #1, all the imposed frequencies were clearly identified and no ghost frequencies were detected, as expected. For test #2, the imposed frequencies in Y direction (2, 5 and 7 Hz) are also identified in X direction having small magnitudes and being classified as ghost frequencies. It is due to the rotations applied to the camera sensor plan. In test #3 one can observe that Y direction frequencies have great frequency magnitudes also in X direction due to significant values of rotations. Ghost frequency are also observed in test #3.
3.2 *Actual video*

The cantilever beam presented in figure 5 was dynamically tested and its free vibration response was recorded with a digital camera. Due to equipment limitations (low frame rate), only the first structural natural frequency was excited. The black circle target was placed on the beam free extremity as shown in figure 6 and the structural free vibration video acquisition was made during 10 seconds having a frame rate equals to 30 frames per second in VHS format (images with 640 lines times 480 columns). Figure 6 is also the target view from the camera position during the test.

Using strain-gage measures, the first structural natural frequency was identified as 5.6 Hz.

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**Figure 4** - Frequency responses for synthetic tests.

**Figure 5** - The tested beam.

**Figure 6** - Target placed on the beam free extremity.
Figure 7 presents the identified time histories in X and Y directions and their respective frequency responses. The natural frequency of the beam (5.6 Hz) is clearly identified in frequency domain regarding Y direction response. The frequency response in X direction also presents the natural frequency of 11.2 Hz (two times 5.6 Hz) as expected for this structure. The accuracy of frequency identification was not significantly affected by the camera position as one may observe in figure 7. It was due to the small rotations $\theta$, $\varphi$ and $\kappa$ of the experimental test.

4 CONCLUSIONS:

A simple method applied to structural frequency identification was presented in this paper. Using a standard digital camera, the displacement time history of a tested structure is obtained through image processing and natural vibration frequencies are identified by means of Fourier Transform. Despite of its limitations presented in section 2.3, the results obtained in the experimental test were considered satisfactory.

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6 REFERENCES:

